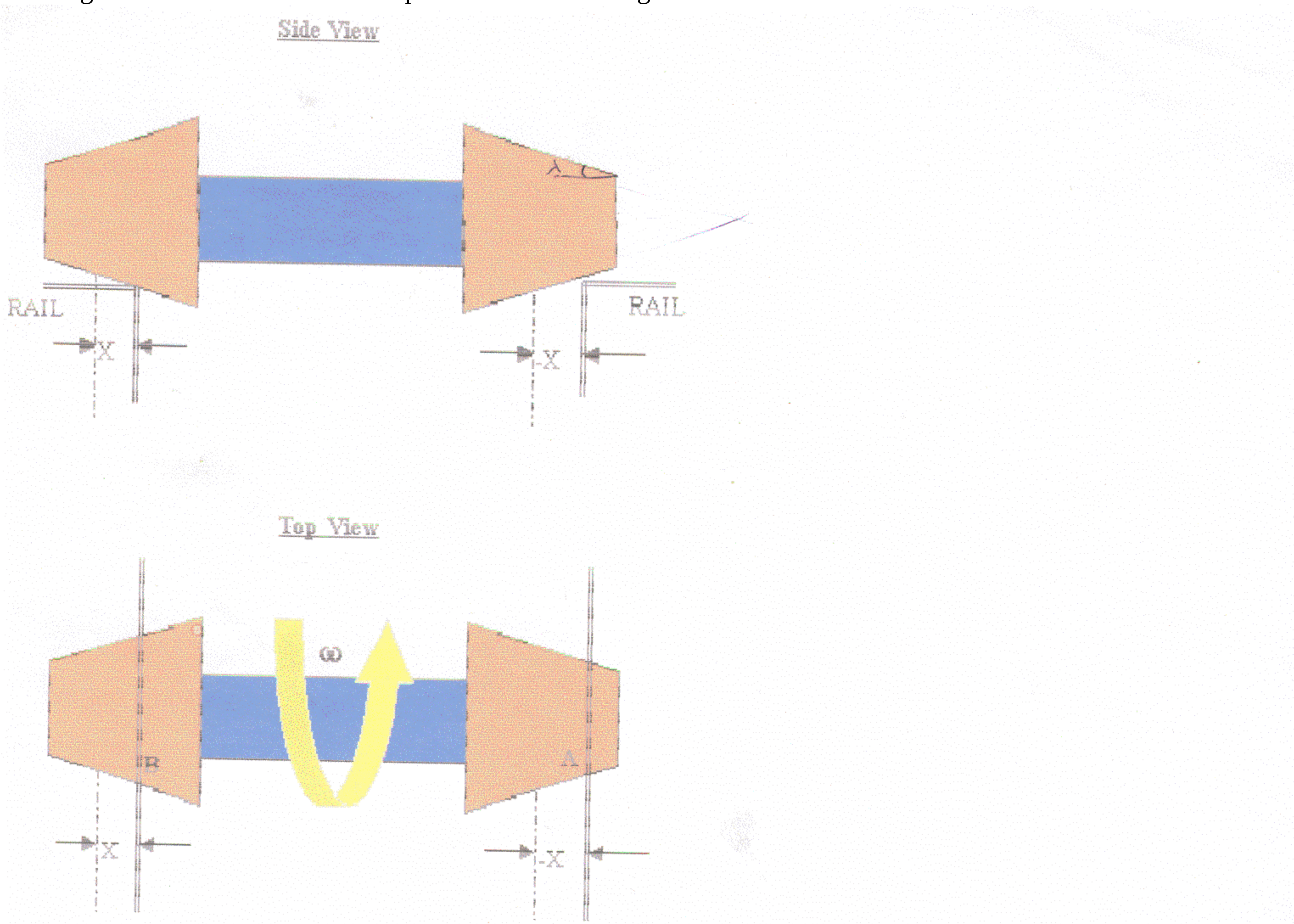
Rail Wheel Hunting

Lets begin with a conical wheel on a perfect track. The diagram is shown below



In the above the following is assumed

1. Conical flangeless wheel. Thus there would be no flanging or flange touching the rail. If the value of X, the lateral displacement of the wheel from centre, in the above diagram is not high, this assumption is redundant.
2. Zero angle of attack (i.e. angle between rail and perpendicular to wheel-set centre line). This simplifies the calculations as linear and angular velocity are uni-directional.
3. Rolling without slipping at constant angular velocity of ω. This assumption ensures that an exact relationship between angular velocity and linear velocity exists.
4. Angle of taper (or conicity) is λ. This is assumed to be constant. In case of new IRS wheels this assumption is true if rail-head is assumed flat & horizontal, as shown in the above diagram.
5. Nominal radius is Ro and dynamic gauge is Lg.

From the above,

Ra, Radius at point A -the right rail-wheel contact point,

= Ro-X*tanA.

Similarly,

Rb, Radius at point B -the left rail-wheel contact point,

= Ro+ X*tanA.

Hence,

Rb-Ra = 2X*tanA. ...[1]

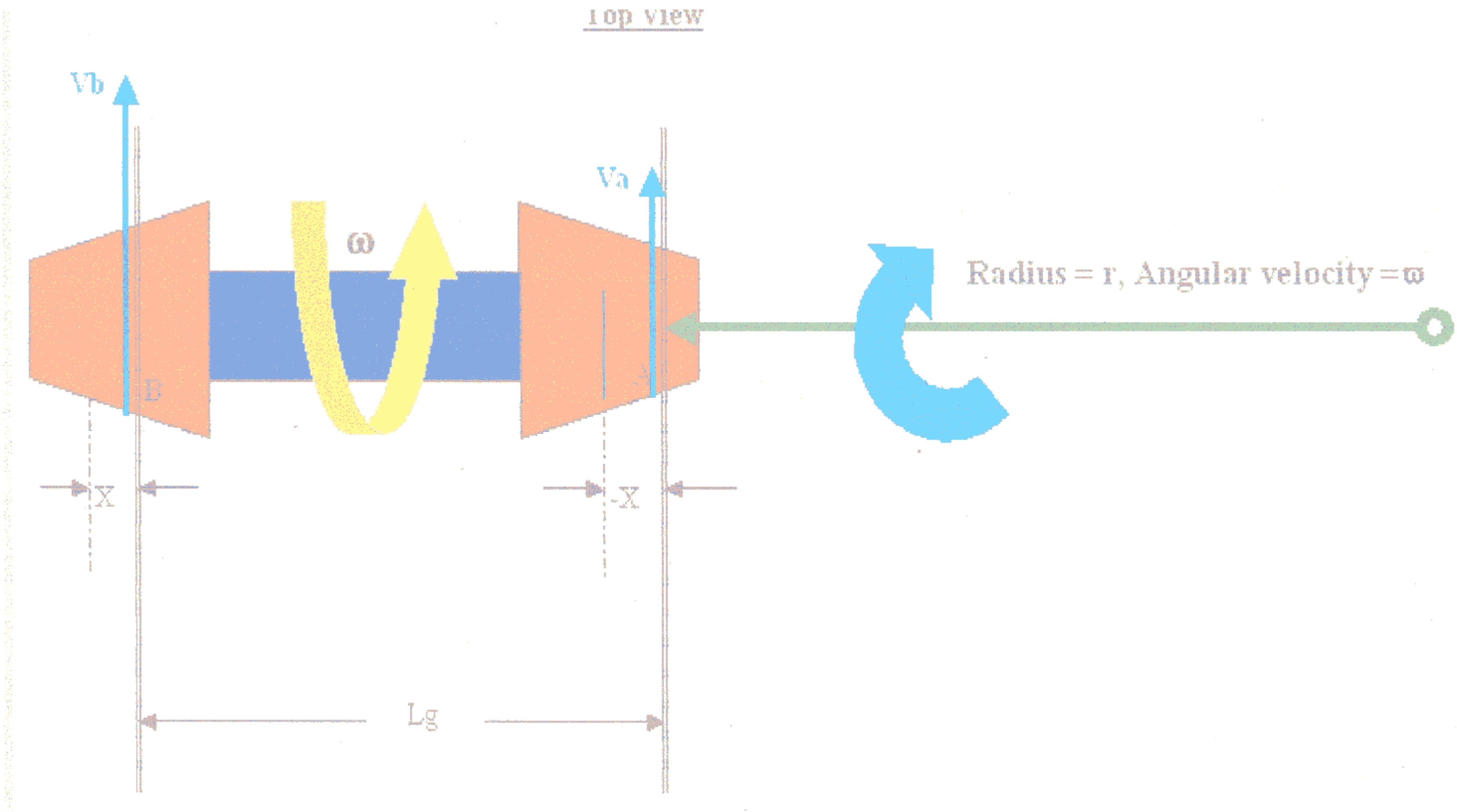
Therefore, Va, linear velocity at A

Va = Ra*ω ...[2]

Similarly, Vb, linear velocity at B ,

Vb = Rb*ω …[3]

In both the above case the linear velocity is along the track, as rotation is perpendicular to track. Since the linear velocity is more at B than A, the axle would turn right with instantaneous angular velocity ϖ and radius 'r' as shown in the diagram shown below:



Hence,

Vb = ϖ*(r+Lg) ...[4]

Va = ϖ * r ...[5]

From [4 & [5],

Vb/Va= 1+Lg/r …[6]

From [2] & [3],

Vb/Va=Ra/Rb …[7]

[7]=[6]:

Rb/Ra= 1+Lg/r

or,

(Rb-Ra)/Ra= Lg/r

r = (Ra*Lg)/(Rb-Ra) …[8]

From [2] & [5],
$Va = \varpi * r = Ra * \omega$

or,

$$\varpi = Ra * \omega / r$$

putting [8] in above,

$$\varpi = Ra\ (Rb-Ra) * \omega / (Ra*Lg)$$

$$= \omega*(Rb-Ra)/Lg$$

putting [l] in above,

$$\varpi = 2X/Lg * \omega \tan \lambda \qquad ...[9]$$

N ow ,since the angle of attack is zero & rate of change of magnitude of ϖ is small, ,instant radial acceleration is centripetal acceleration $a = - \varpi^2 r$

$$= - [\omega*(Rb-Ra)/Lg]^2 * Ra*Lg/(Rb-Ra)$$

$$= - \omega^2*Ra(Rb-Ra)/Lg$$

$$= - 2\omega^2*Ra* X*\tan\lambda /Lg$$

$$a = - [2\omega^2*Ra *\tan\lambda /Lg] * X$$

Now, $\tan \lambda=1/20$ for IRS profile, Ro=.5m and if X is small %changes in Ra can be ignored. Thus, if $B = [2\omega^2*Ra *\tan\lambda /Lg]$, then

$$a = d^2X/dt^2 = - B*X \qquad …[10]$$

The above differential equation is for simple harmonic motion $x= A\sin\Phi t$,where $\Phi=2\pi *f$, f is the natural frequency. Therefore, $dx/dt =A(f)\cos\Phi t$ & $d^2X/dt^2= -A\Phi^2\sin\Phi t$. Thus, equation [10] becomes

$$-A\Phi^2\sin\Phi t= -B* A\sin\Phi t$$

or,

$$\Phi^2= B$$

,

or,

$$\Phi= \sqrt{B}$$

therefore,

$$f= (\sqrt{B})/(2\pi)$$

Now, speed of simple harmonic motion is wavelength*frequency. Speed $=Ro*\omega$ & $Ra \approx Ro$.

Therefore,

$$\text{wavelength}* f =Ro*\omega$$

$$\text{wavelength} = Ro*\omega/f = 2\pi Ro*\omega/(\sqrt{B})$$

$$= 2\pi Ro*\omega/\sqrt{[2\omega^2*Ra*\tan\lambda. /Lg]}$$

Hence, wavelength = $2\pi\sqrt{[Ro* Lg /2\tan\lambda.]}$